Noise-induced breakdown in linear self-driven particle systems

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1. Collective dynamics by active particles

- Self-organized collective phenomenona in animal swarms, fish schools, pedestrians, robots, colloids, bacteria, road traffic, ...
- Motility Induced Phase Separation: positive feedback between aggregation and slowing down
- Moving bands in flocking models, Active Ising models, charged colloids
- Succession of de- and acceleration: stop-and-go waves in pedestrian, vehicular and bicycle traffic

2. Stop-and-Go: Deterministic Models

- Instability of the homogeneous configuration
- Phase transition & fine tuning of parameters, limit-cycle solution only for non-linear models
- E.g. Optimal-Velocity model $\ddot{x}_i = \frac{1}{\tau}(V(\Delta x_i) - \dot{x}_i)$ linearly stable for $\tau > 1/(2V')$

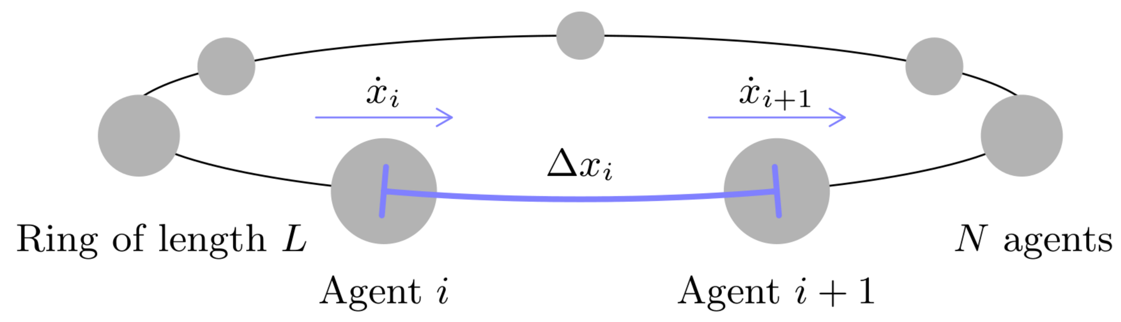


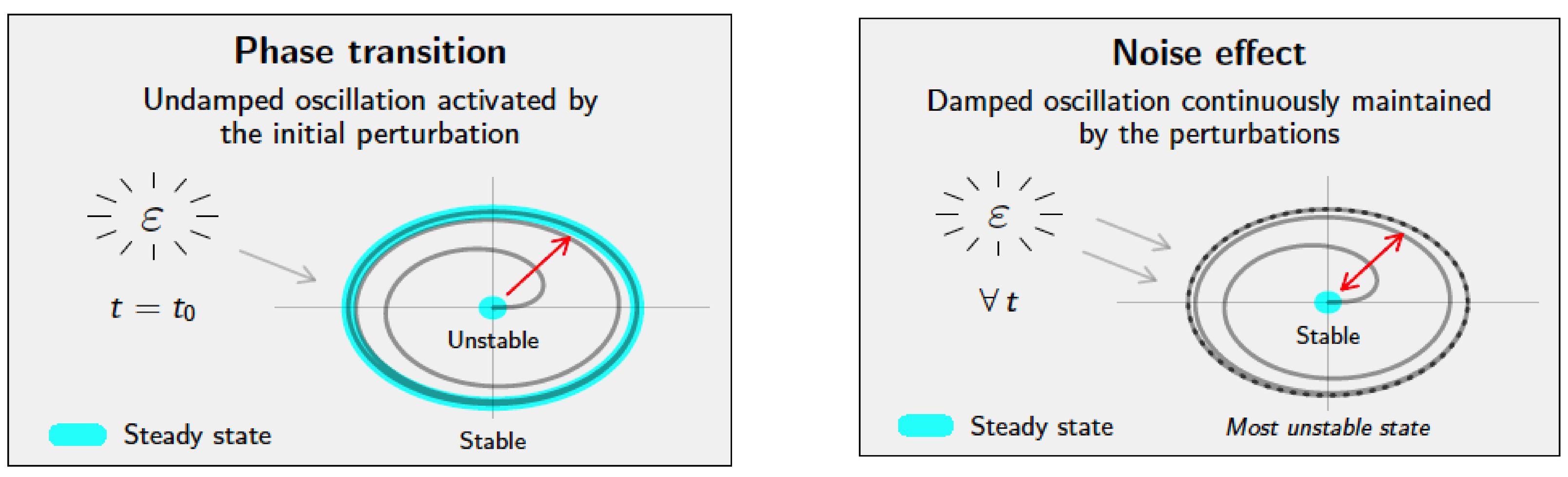
Figure 1: Scheme for the considered system of N particles on a ring of length L.

3. Stop-and-Go: Stochastic Models

- The stochastic microscopic motion model is

$$\begin{cases} dx_i = [\lambda(\Delta x_i - \ell) + z_i]dt, \\ dz_i = [\gamma(\Delta x_i - \Delta x_{i+1}) - \beta z_i)]dt + \sigma dW_i, \end{cases} \qquad (1)$$

where the $W_i(t)$ are independent Wiener processes
- Purely linear, stochastic differential system: Minimal deterministic part and stochastic Gaussian noise
- No instability, phase transition, or non-linearity needed for phase separation → Noise Induced Stop-and-Go dynamics!



(a) (b)

Figure 2: Illustrative scheme for the modeling of stop-and-go dynamics with phase transition in the periodic solution (a) and the noise-induced oscillating behavior (b).

4. Results

- The sufficient stability condition is

$$-\beta\lambda < 2\gamma < \beta\lambda \qquad (2)$$

- change dynamics from persisting waves ($\gamma > 0$; reaction rate), to dissipating waves ($\gamma < 0$; anticipation rate)
- Under stability exactly solvable: the model is a Gaussian process with a single stationary distribution
- Expectation tends to space-homogeneous solution $\mathbb{E}[\mathcal{X}(t)] \to 0$ as $t \to \infty$
- Stop-and-Go waves as non-trivial fluctuations in co-variance matrix $\Sigma(\infty)$
- Linear stability can not ensure the absence of jams!

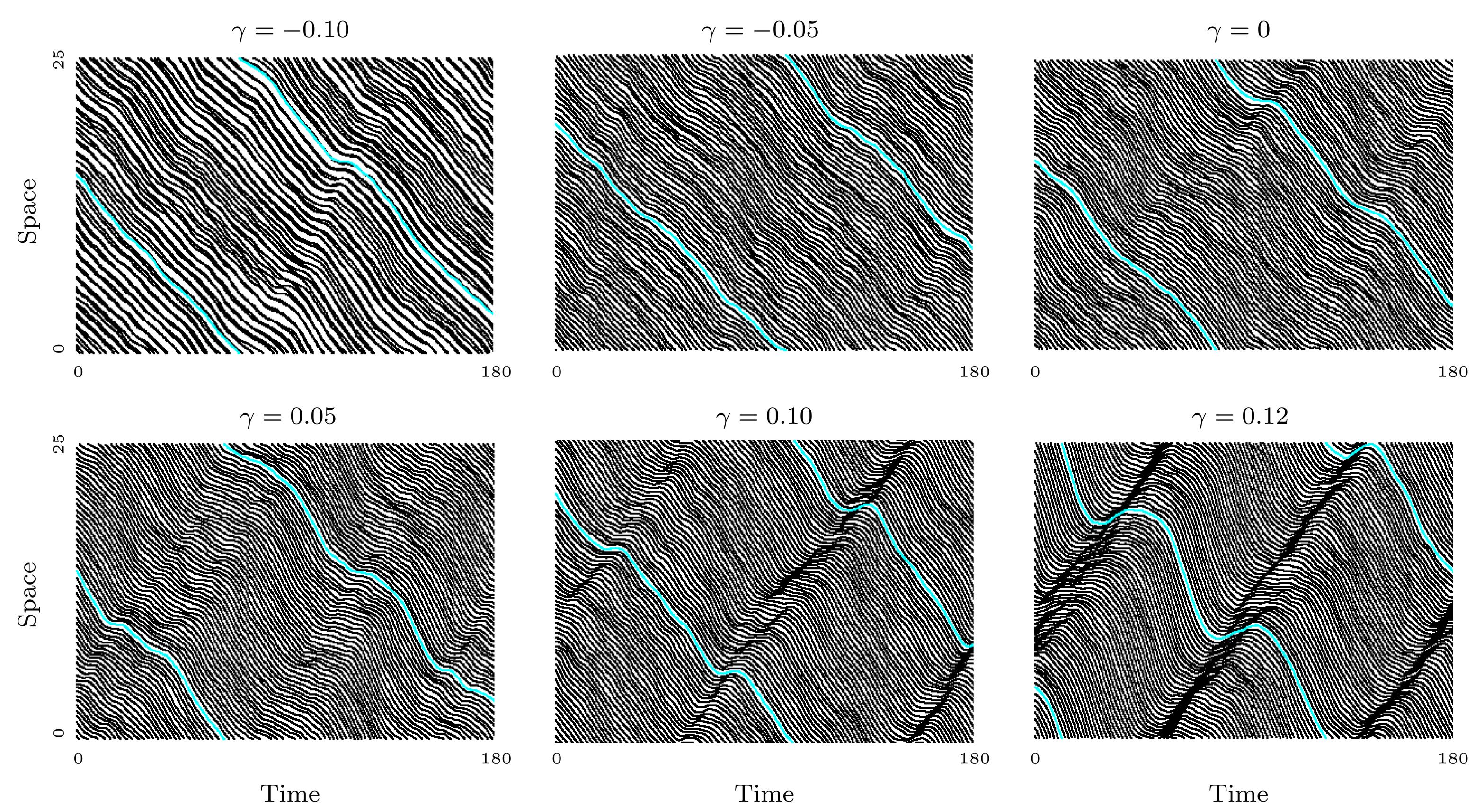


Figure 3: Examples of trajectories of 50 particles in stationary states for $\lambda = 1$, $\beta = 0.2$, and different values of γ. The dynamics range from 2 by 2 coupled configurations as γ tends to γ_c^1, homogeneous configurations for moderated negative values for γ, to configurations with a single stop-and-go wave as γ tends to γ_c^2. Note that the trajectories with no noise ($\sigma = 0$) are systematically strictly homogeneous (i.e. parallel and equidistant).

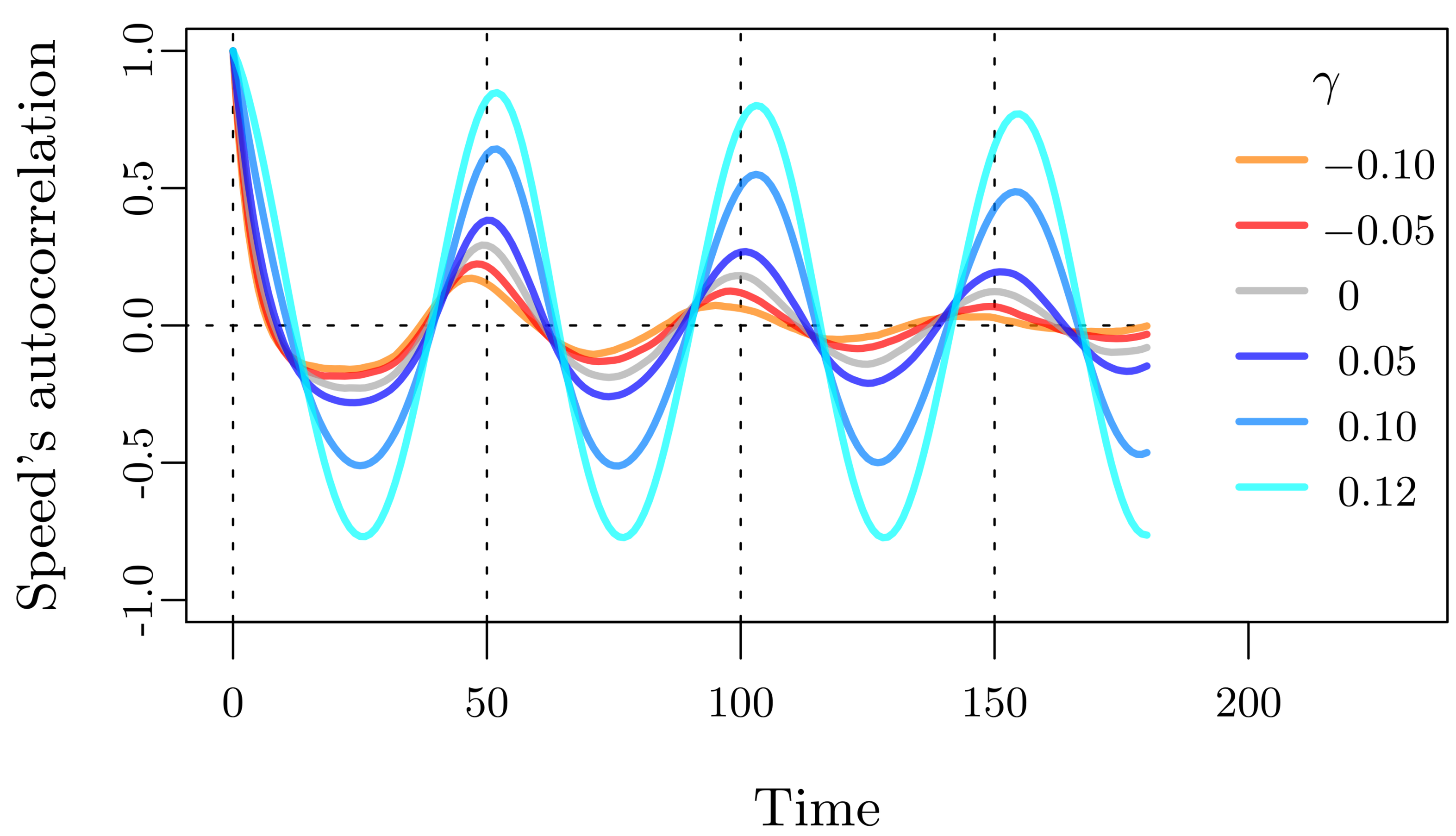


Figure 4: Auto-correlation in time of a single vehicle in stationary state for different stable values for the parameter $\gamma = -0.1, 0.05, 0, 0.05, 0.1, 0.12$ (the stability condition being $\gamma_c^1 = -0.1 < \gamma < \gamma_c^2 \approx 0.1285$). The autocorrelation functions oscillate, especially as γ tends to γ_c^2. This attests for the presence of periodic stop-and-go dynamics.

References

1. Bando, M., Hasebe, K., Nakayama, A., Shibata, A. & Sugiyama, Y. Dynamical model of traffic congestion and numerical simulation. *Phys. Rev. E* **51**, 1035–1042 (1995). 2. Cates, M. E. & Tailleur, J. Motility-Induced Phase Separation. *Annual Review of Condensed Matter Physics* **6**, 219–244 (2015). 1406.3533. 3. Friesen, M., Gottschalk, H., Rüdiger, B. & Tordeux, A. Spontaneous Wave Formation in Stochastic Self-Driven Particle Systems. *SIAM Journal on Applied Mathematics* **81**, 853–870 (2021). 4. Tordeux, A. & Schadschneider, A. White and relaxed noises in optimal velocity models for pedestrian flow with stop-and-go waves. *Physics A* **18**, 185101 (2016). 5. Vicsek, T. & Zafeiris, A. Collective motion. *Physics Reports* **517**, 71–140 (2012). 1010.5017. 6. Vissers, T., van Blaaderen, A. & Imhof, A. Band Formation in Mixtures of Oppositely Charged Colloids Driven by an ac Electric Field. *Physical Review Letters* **106**, 228303 (2011). 7. Zhang, J. *et al.* Universal flow-density relation of single-file bicycle, pedestrian and car motion. *Physics Letters A* **378**, 3274–3277 (2014).